

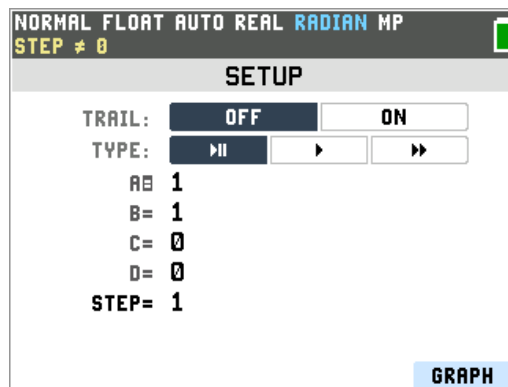
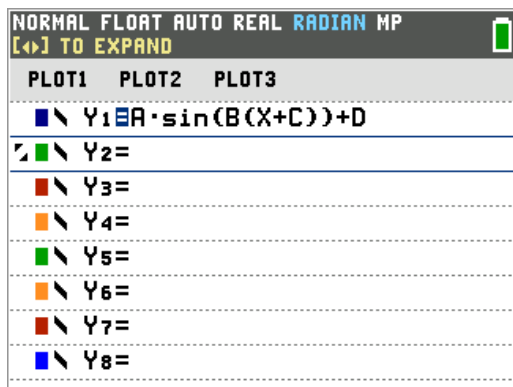
Sinusoidal Function Transformations

Introduction

Additive and multiplicative transformations of sinusoidal functions occur frequently in practical applications. For example, seasonal temperature patterns, tides and ocean water levels, and the phases of the moon might all be modeled by a sinusoidal function.

In this activity, we will examine the role of the constants A ($A \neq 0$), B , C , and D in functions of the form $f(x) = A \sin(B(x + C)) + D$. Let's start by looking at the sine function and additive transformations. Then we will consider multiplicative transformations to the sine function, and eventually combine all of these results.

The problems in this activity involve the Transformation application. Begin by entering the function $y_1 = A \sin(B(x + C)) + D$ in the $y=$ editor. Press $\boxed{\text{window}}$ and set $X_{\min} = -4\pi$, $X_{\max} = 4\pi$, $Y_{\min} = -8$, and $Y_{\max} = 8$. Press $\boxed{2\text{nd}}$ $\boxed{\text{format}}$ and select to turn on the grid lines. Press $\boxed{\text{on}}$ to return to the Home Screen. From the Home Screen, select the Transformation Graphing App. Use the Options; Setup menu (in the $y=$ menu) to set the initial values of the constants $A = 1$, $B = 1$, $C = 0$, and $D = 0$. Note that the default step for each variable is set to 1.



Problem 1: Vertical Translations

In this problem we compare the graph of the function f defined by $f(x) = \sin x$ and the graph of g defined by $g(x) = \sin x + D$.

On the graph screen, the Transformation Graphing App initially displays a graph of the function $y = A \sin(B(x + C)) + D = 1 \cdot \sin(1 \cdot (x + 0)) + 0 = \sin x$.

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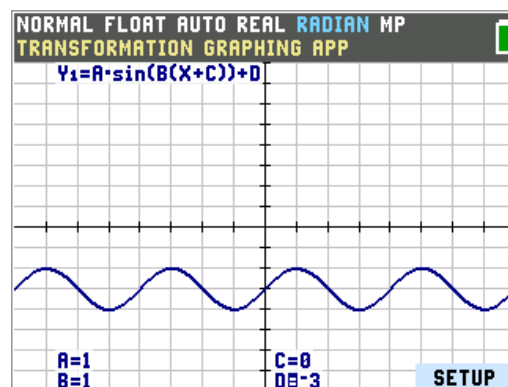
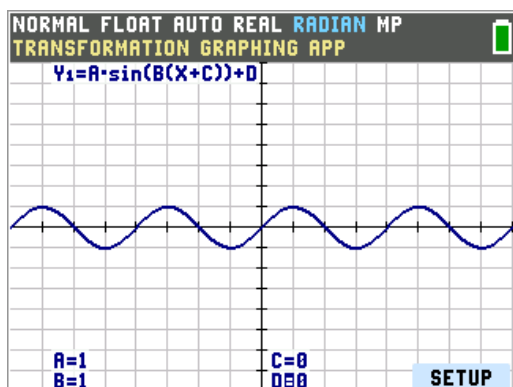
Arrow down to the variable D . Use the left and right arrows to step through various values for D and observe how the graph changes. Describe the relationship between D and the graph's behavior.

Consider two specific values of D .

Describe how the graph of the function $y = \sin x + 2$ is related to the graph of $y = \sin x$.

Describe how the graph of the function $y = \sin x - 3$ is related to the graph of $y = \sin x$.

Describe in general how the graph of $y = \sin x + D$ is related to the graph of $y = \sin x$.



Problem 2: Horizontal Translations

In this problem we compare the graph of the function f defined by $f(x) = \sin x$ and the graph of g defined by $g(x) = \sin(x + C)$.

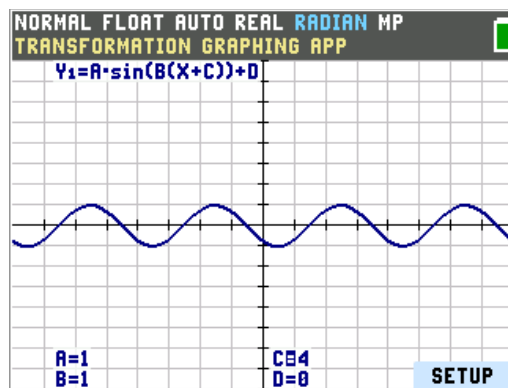
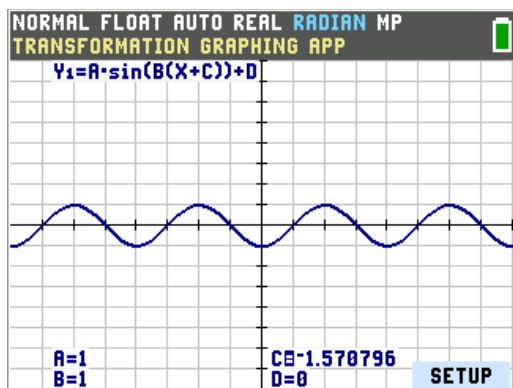
In the Transformation Graphing App, set $D = 0$. Arrow up to the variable C . Use the left and right arrows to step through various values for C and observe how the graph changes. Describe the relationship between C and the graph's behavior.

Consider two specific values of C .

Change the value of C to describe how the graph of $y = \sin\left(x - \frac{\pi}{2}\right)$ is related to the graph of $y = \sin x$.

Change the value of C to describe how the graph of $y = \sin(x + 4)$ is related to the graph of $y = \sin x$.

Describe in general how the graph of $y = \sin(x + C)$ is related to the graph of $y = \sin x$.



Problem 3: Vertical Dilations

In this problem we compare the graph of the function f defined by $f(x) = \sin x$ and the graph of g defined by $g(x) = A \cdot \sin x$.

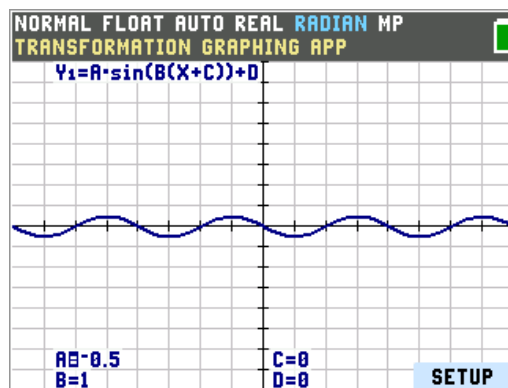
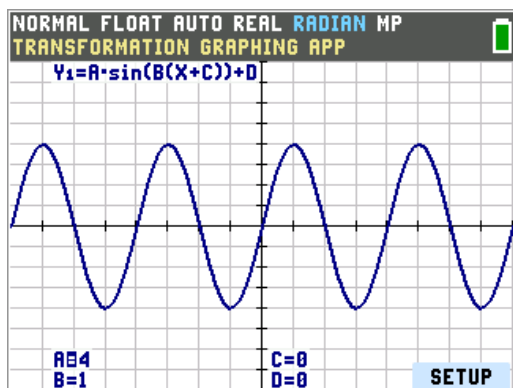
In the Transformation Graphing App, set $C = 0$ ($A = 1, B = 1, D = 0$).

Use the up/down arrow to move to the variable A . Use the left and right arrows to step through various values for A and observe how the graph changes. Describe the relationship between A and the graph's behavior.

Change the value of A to 4 and describe how the graph of $y = 4 \sin x$ is related to the graph of $y = \sin x$.

Change the value of A to $-\frac{1}{2}$ and describe how the graph of $y = -\frac{1}{2} \sin x$ is related to the graph of $y = \sin x$.

Describe in general how the graph of $y = A \sin x$ is related to the graph of $y = \sin x$. Make sure to explain the case where $A < 0$.



Problem 4: Horizontal Dilations

In this problem we compare the graph of the function f defined by $f(x) = \sin x$ and g defined by $g(x) = \sin(Bx)$.

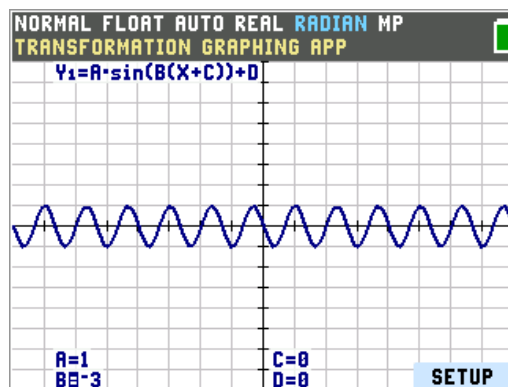
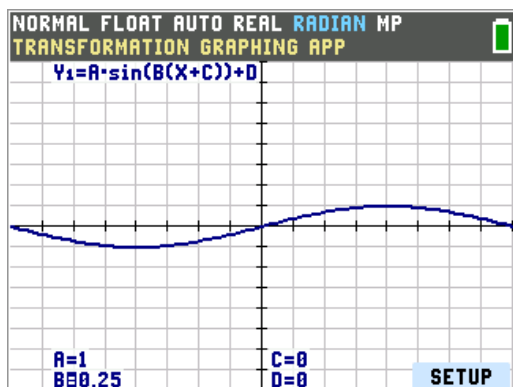
Set $A = 0$, $B = 0$, $C = 0$, and $D = 0$.

Use the up/down arrows to move to the variable B . Use the left and right arrows to step through various values for B and observe how the graph changes. Describe the relationship between B and the graph's behavior.

Change the value of B to $\frac{1}{4}$ and how the graph of $y = \sin\left(\frac{1}{4}x\right)$ is related to the graph of $y = \sin x$.

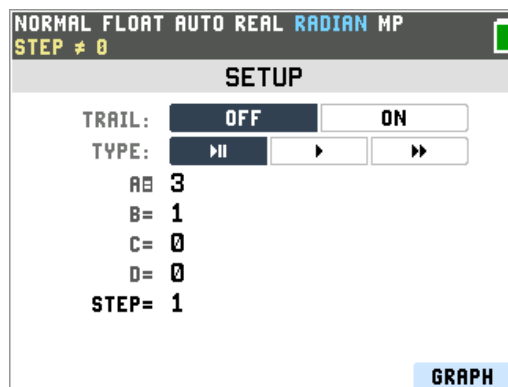
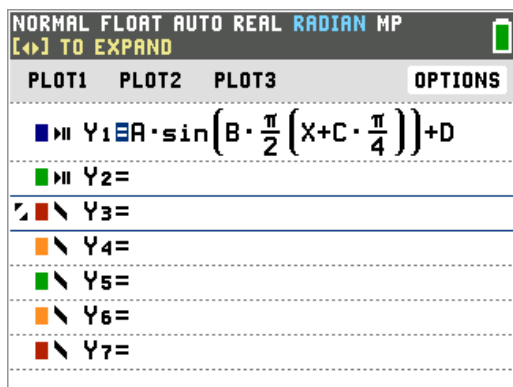
Describe how the graph of $y = \sin(-3x)$ is related to the graph of $y = \sin x$.

Describe in general how the graph of $y = \sin(Bx)$ is related to the graph of $y = \sin x$.



Challenge

Enter the function $Y_1 = A \sin\left(\left(B \cdot \frac{\pi}{2}\right)\left(x - C \cdot \frac{\pi}{4}\right)\right) + D$ in the $\boxed{Y=}$ editor. In the Transformations Graphing App set $A = 3$, $B = 1$, $C = 0$, and $D = 0$.



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On the graph screen, use the left and right arrows to step through various values for B and observe how the graph changes. Describe the relationship between B and the graph's behavior.

Set the value of $B = \frac{2}{\pi}$. Use the left and right arrows to step through various values for C and observe how the graph changes. Describe the relationship between C and the graph's behavior.

Summary

Combine the results about additive and multiplicative transformations of the sine function. Let the function f be defined as $f(x) = A \sin(B(x + C)) + D$, $A \neq 0$. Fill in the blanks.

(1) $|A|$ is the amplitude of the graph of f . If $A < 0$, then the graph of f is also reflected over _____. Remember that the amplitude is always positive, but the constant A can be positive or negative.

(2) The graph of f is a _____ of the graph of the sine function by a factor of _____. The graph of f has period _____. If $B < 0$, then the graph of f is a reflection over _____.

Note that it may be necessary to factor the argument of the sine so that the input expression is in the form $B(x + C)$. Then we can identify the constants B and C .

(3) The value of C is the phase shift. If $C > 0$, then the graph is shifted _____. If $C < 0$, then the graph is shifted _____.

(4) The value of D is the midline. If D is positive the graph is shifted _____. If D is negative the graph is shifted _____.

Exercises

(1) Find the amplitude, period, phase shift, and vertical shift of the function, and sketch the graph of the function.

(a) $f(x) = -2 \sin(3(x + \pi))$

(b) $f(x) = 6 \sin(\pi x) - 2$

(c) $f(x) = 2\pi \sin(-3(x + 1)) + 4$

(d) $f(x) = -2 \sin\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) + 3$

(2) Find a function f of the form $f(x) = A \sin(B(x + C)) + D$ with the given period, phase shift, amplitude, and vertical shift.

(a) Period 3, phase shift 0, amplitude 5, vertical shift -4

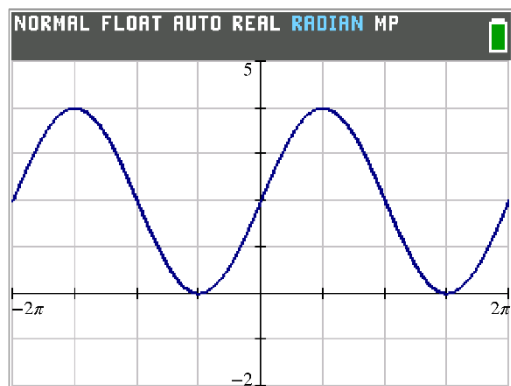
(b) Period 4π , phase shift 1, amplitude 2, vertical shift 0

(c) Period $\frac{1}{2}$, phase shift $-\pi$, amplitude 3, vertical shift 3π

(d) Period 6, phase shift 3, amplitude 2, vertical shift -3

- (3) For each graph of the function f of the form $f(x) = A \sin(B(x + C)) + D$, find the values of A , B , C , and D .

(a)



(b)

